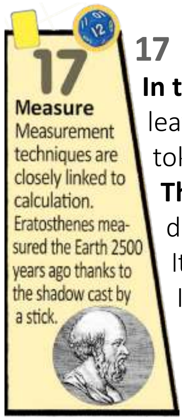


17

In the game: If the player answers the question correctly, they may leave their piece on this box and gain a dodecahedral “Roll the die” token, if present on the table.

The number: • It is the smallest prime which is the sum of two distinct fourth powers: $17 = 1^4 + 2^4$ • $17 = 3^4 - 4^3$ • $17 = 2^3 + 3^2$ • It is the smallest prime whose sum of digits is a cube: $1 + 7 = 8 = 2^3$ • It is the only prime which is the sum of four consecutive primes: $17 = 2 + 3 + 5 + 7$ • 17 is prime and 71 is also prime • Friday 17 has been considered unlucky in Italy since time immemorial, probably because of the inscription on the Roman tombs VIXI ("vissi") composed of the letters VI and XI, which when interpreted as numbers have a sum of 17: the roads to superstition are infinite… • Among the first 100 million primes, those ending with 17 are more frequent than those ending with any other pair of digits • If a nine is inserted between the digits of the twin primes

17 and 19, we obtained a pair of three-digit twin primes: 197 and 199 • $17 + 2^1$; $17 + 2^1 + 2^2$; $17 + 2^1 + 2^2 + 2^3$; $17 + 2^1 + 2^2 + 2^3 + 2^4$ and $17 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$ are primes • There are no consecutive primes that differ by 17 • The expression $n^2 + n + 17$ is peculiar: by assigning to n any value from 0 to 15 we obtain a prime number between 17 and 257 • The sum of the numbers in the cube of 17, which is 4,913, is 17; indeed, $4 + 9 + 1 + 3 = 17$ • A number with more than two digits is divisible by 17 if the number obtained by eliminating the units digit minus the quintuple of the units digit is 0, 17 or a multiple of 17.
For example: 3,026 is divisible by 17 if the number $302 - (5 \times 6) = 272$ is. Repeating the operation, 272 is divisible by 17 since the number $27 - (5 \times 2) = 17$. Thus 3,026 is too. •

Measurement: Measurement, or measure, is the attribution of a value to a particular physical or chemical property of an object or substance. This value makes reference to a scale that is assumed to include all the values that the property can take, which may have upper or lower bounds or be infinite. For example, on the temperature scale there is a lower limit, which is -273.15 °C (absolute zero), while for distances or other quantities it can be conceived that there is no limit.

Measurements can be obtained through many procedures. In the case of the length of objects, it can consist of a simple comparison with another object of, or marked with, standard lengths. This is the operation we perform when we measure the length of a line segment with a ruler, where the centimetres or inches marked on the ruler are the standard lengths.

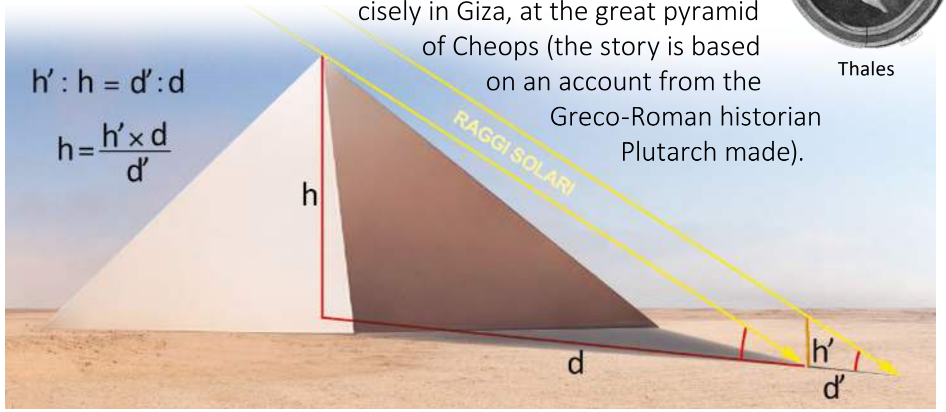
The science of defining and studying unit of measurement is metrology, in which mathematics plays an important role. Also because The size of certain units has varied over the ages and varies from culture to culture, despite efforts to use a universal system. For example, the whole system is managed with metric length units in half the world (the metre and its multiples and subdivisions) and with imperial units in the other half (the inch, foot, yard, and mile). Thus, mathematics is indispensable for carrying out conversions from one unit to another. To see that this issue is very serious, we need only think of the plane crashes that have occurred due to an incorrect conversion between metric and imperial units or a lack of familiarity in the use of a certain unit by personnel accustomed to using a different one.

It is more interesting to consider how we can measure a property of an object that we cannot approach, or which is too large to be physically compared with a sample. This is called remote measurement or telemetry. Let's see some historical cases of telemetry that show us how we can better come to know the reality that surrounds us even with rudimentary tools.

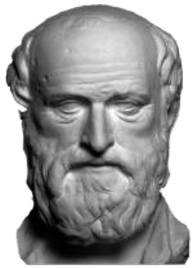
Imagine following the Greek philosopher, mathematician and engineer Thales

of Miletus (624 - 546 B.C.) on his journey to Egypt. Thales was one of the first thinkers who attempted to explain natural phenomena without resorting to myths, the first exponent of naturalistic or materialistic philosophy. Here he reaches the Nile valley precisely in Giza, at the great pyramid of Cheops (the story is based on an account from the Greco-Roman historian Plutarch made).

Thales



At the time, this was already an ancient monument, of which nobody knew the height. Thales, questioned on this subject by none other than Pharaoh, immediately found a way to calculate this value. He planted a stick in the ground and measured its shadow and that of the pyramid. Let's allow Plutarch to tell the rest:



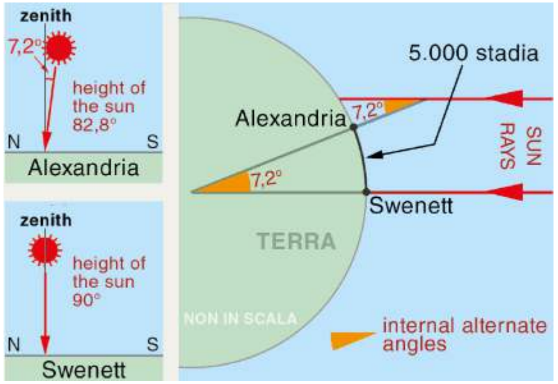
Eratosthenes

...without the aid of tools, a stick was planted at the edge of the shadow projected by the pyramid, since the sun's rays, striking the stick and the pyramid, formed two triangles, [he showed] that the height of the rod and that of the pyramid are in the same proportion as their shadows.

Obviously, Thales knew the properties of similar triangles; perhaps he had discovered them himself.

Now we remain in the Nile valley, but we move on three centuries to meet Eratosthenes (276 - 194 B.C.), born in Cyrene, mathematician, astronomer, geographer, as well as director of the Library of Alexandria and also poet.

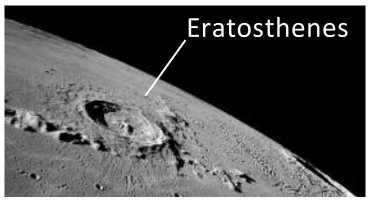
Alongside is a map of the world as he knew it and described it, from a French printing press in the mid-nineteenth century.



Eratosthenes explains how he measured the size of the planet Earth. He knows that in Swenett (the city known to the Greeks as Suene and known today as Aswan), at midday on the day of the summer solstice the sun is at its zenith, that is, it has an angle of 90° compared with the horizon. Thanks to a stick stuck in the ground in Alexandria, located north of Swenett, at the same time on the same day, he determined that the sun had an angle from the horizon of 82° 48', which is 7° 12' = 7.2° (one fiftieth of a circle, since 7.2 = 360/50) lower than in Swenett.
Eratosthenes knew well that the Earth is (approximately) spherical and that if a straight line crosses two parallel lines, the resulting internal alternate angles are equal. The angle between the two measurements of the height of the sun therefore correspond, at the centre of the Earth, to the angle between the vertical lines passing through the two locations. In other words, the 7.2° circle arc between Alexandria and Swenett is 1/50 of the circumference of the Earth. Knowing that the distance between Alexandria and Swenett is 5,000 stadia (1 Egyptian stadium = 157.5 meters), the simple equation of ratios

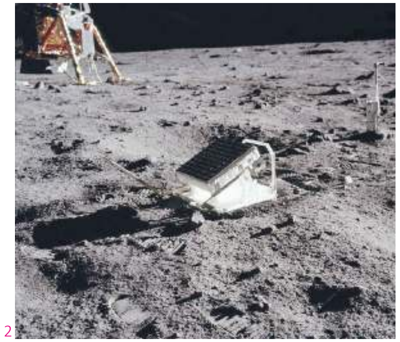
$$7.2 : 360 = 5000 : x$$

allowed him to calculate that the circumference of the Earth is 250,000 stadia, equal to 39,375 km, a value amazingly close to the actual value of 39,941 km of the polar circumference (the one passing through the poles) as we know it today. Here, a free intellect, not contaminated by myths, superstitions, or prejudices, using only the observation of natural facts and elementary experimentation, took humanity forward a giant step, managing to measure the dimensions of the planet on which we stand. Eratosthenes seems to have also found ways to measure the distance between the Earth and the Sun and between the Earth and the Moon, but in these matters the history is fragmented. To honour his memory, an important lunar crater was named after him.
Speaking of telemetry and the Moon, we now see how the problem of determining the Earth-Moon distance was tackled two millennia after Eratosthenes. We follow the Apollo 11 astronauts in their first steps to the moon in July 1969.

1


They are placing a retroreflector on the surface, which is practically an all-direction

1

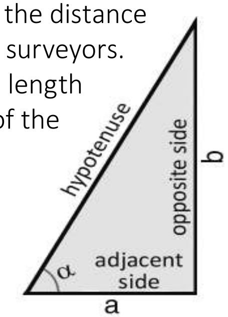

2


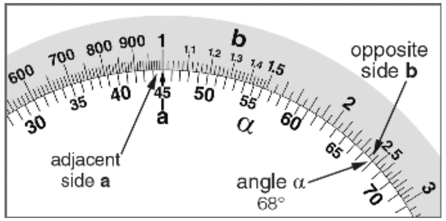
On the left, the laser beam leaves from the McDonald observatory in the direction of the retroreflector. On the right is the retroreflector, placed on the lunar surface.

reflector like the one on the mudguard of a bicycle, as part of the Lunar Laser Ranging experiment.
The operation is simple: a laser beam is sent from the Earth to the retroreflector, which reflects it back to Earth, where it is received; the time taken by the laser signal to go and return is measured (about 2.5 seconds), multiplied by the speed of light (299,792 km per second) and the result is divided by two to obtain the distance from the Earth to the Moon. Suppose that at some point the signal returns after 2.56 seconds. The distance of the Moon at that time is

$$(299{,}792 \times 2.56) / 2 = 383{,}733.76 \text{ km}$$

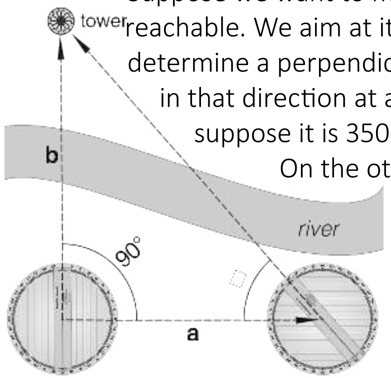
The measured time varies continuously, because the distance between the Earth and the Moon varies continuously. The average distance is 384,467 km. The accuracy of these measurements is in the order of a millimetre and this has made it possible to ascertain that the moon is spiralling away from the Earth at a rate of 38 millimetres per year.
Let's move on to the practicalities of using a pair of measuring instruments known as "Quadrant" and the Azimuthal or Pelorus and a calculation tool, the "Right angle Computer". These tools can be found in Appendix 2, ready to be photocopied, cut out and assembled according to the following simple instructions.
The two quadrants enable the measurement of angles: the quarter-circle in the vertical plane and the full circle in the horizontal plane. Note that instruments for measuring angles have been used for millennia by astronomers and navigators to determine the motion of the stars and predict their position, and to navigate the seas and oceans, but also on land, to determine the distance of an unreachable object or its size, which is one of the works of surveyors.
The Right Angle Computer instead allows you to determine the length of the Opposite side of a right-angled triangle (**b**) if the length of the Adjacent side (**a**) and the size of the angle α between the latter and the Hypotenuse is known.
The result can also be obtained by making calculations, but the ruler gives an approximate solution to the problem in two or three seconds.





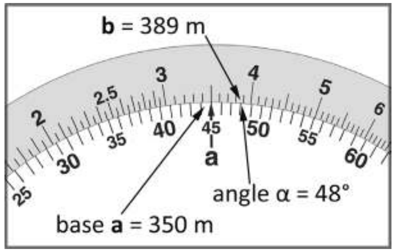
Let's see how it works using the example shown on the right. The arrow marked "**a**" is directed toward the value of the distance a (1 km in the example); at the angle α (68° in the example) the length of the catheter **b** (2.45 km) is read off. Those who know trigonometry can understand that the slide rule produces results based on the formula b = a × tan(α); the external internal scale in fact reports the values of the tangent of the angle α. By setting two of the values, the third is the result. For practical use, however, these theoretical aspects can be ignored.

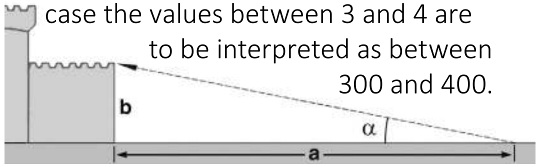
Now let's see some examples of problems that can be solved with these tools.

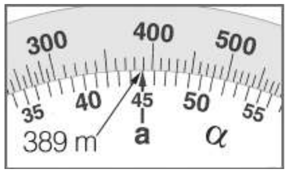


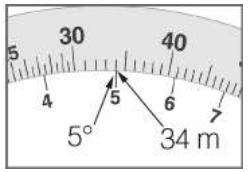
Suppose we want to measure the distance of a tower that is not reachable. We aim at it with the horizontal Quadrant in order to determine a perpendicular (90°) direction. We move the instrument in that direction at a distance a. We have thus defined a "base"; suppose it is 350 m.

On the other end of the base we aim for the tower and determine the angle α; let's say it is for example 48°. By entering the data on the Right Angle Computer, we immediately obtain the length of **b**, which is 389 m. Having used the part of the scale between 1 and 10, the order of magnitude must be correctly evaluated, just like with the slide rule described in an earlier section; in this case the values between 3 and 4 are to be interpreted as between 300 and 400.

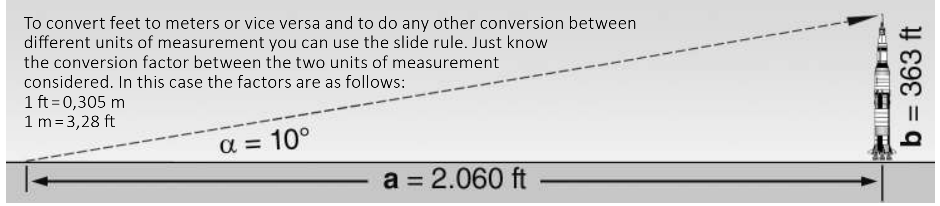


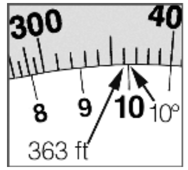


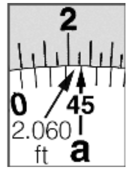




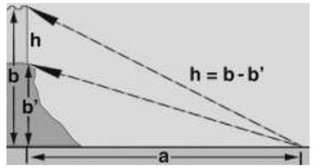
The same goes for the vertical Quadrant. Suppose we want to measure the height of the walls of a castle (below). The distance a is known, perhaps from a measurement with the horizontal Quadrant, and it is 389 m. With the Quadrant we measure an angle α of 5°. With the Ruler, after entering the values as shown here, we find that the walls are 34 m high.

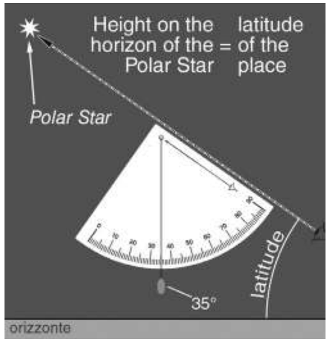






The beauty of these rules is that they allow you to do operations, but also reverse operations. For example, let's say we know the height of an object, say the Saturn V in Cape Canaveral, is 363 feet high (feet, abbreviated ft; we are in the USA so we use the local units of measurement), and we want to determine its distance away from us. In this case we adjust the value of **b**, which is known, to the angle α; then the arrow marked "**a**" indicates the value of **a**, i.e. the distance of the rocket.





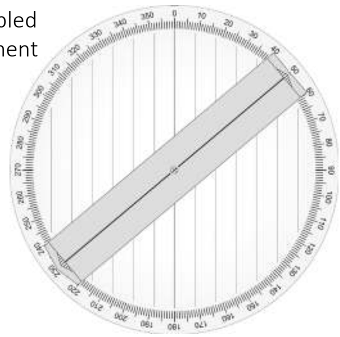
Now let's ask ourselves: what if the object whose height we want to measure does not rest on the horizon, but is placed at a certain height, for example on a hill? No problem, first we measure the height of the upper end **b** of the object, then the height of the lower end **b'**. Just take the difference between the two values to get the height of our object.

Finally, let's not forget that in the Northern hemisphere, the height on the horizon of the Polar Star is equal, up to reasonable accuracy, to the latitude of a place. This notion has been used for ocean sailing for centuries, providing the ship's latitude each night. Navigating in order to keep the Polar at the same height, Columbus and other navigators could proceed around the globe along a predetermined parallel.

APPENDIX 2

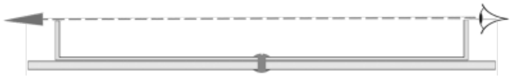
Photocopiable instruments

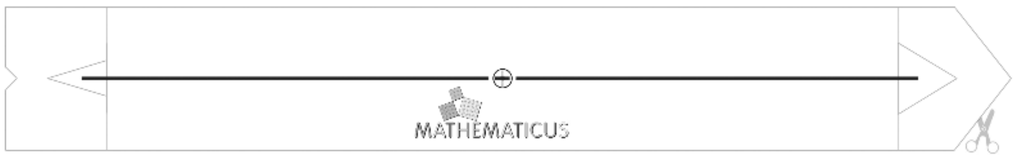
Assembled instrument



The Azimuthal Quadrant or Pelorus

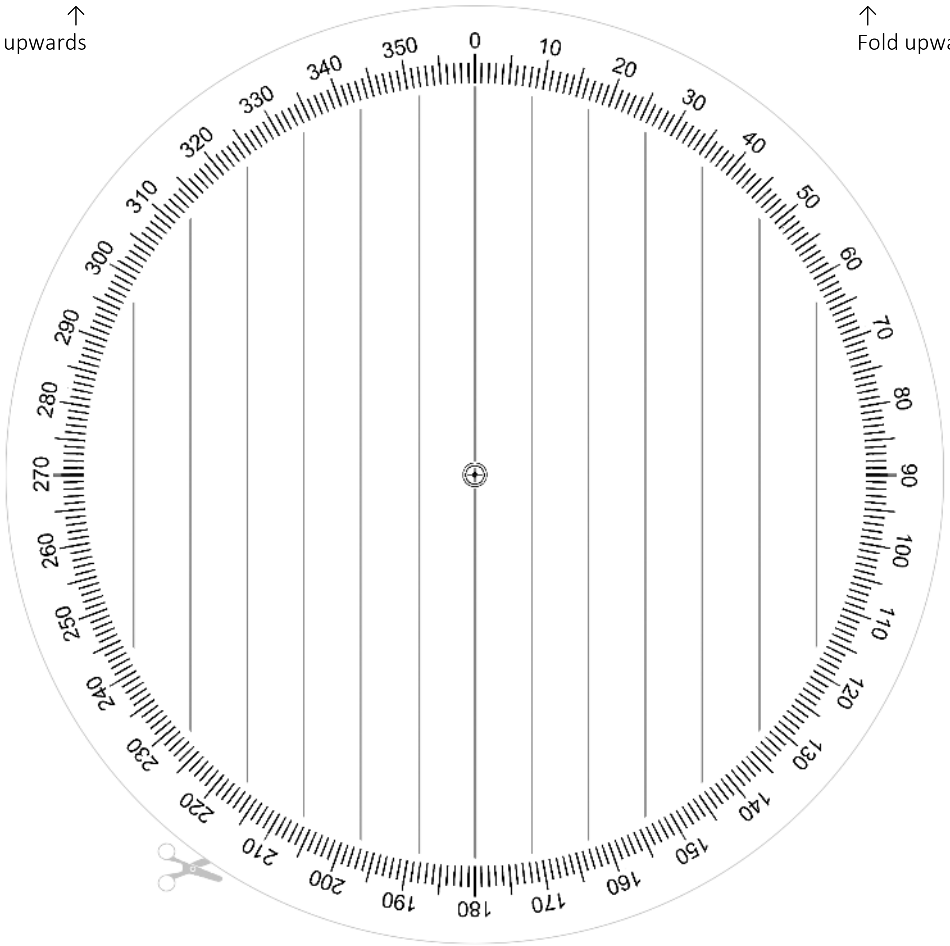
Photocopy and paste the shapes onto a sturdy flat sheet of cardboard and cut them out. Pierce holes and assemble the instrument using a rivet or a paper fastener. The raised parts of the instrument are used to pinpoint the desired direction.





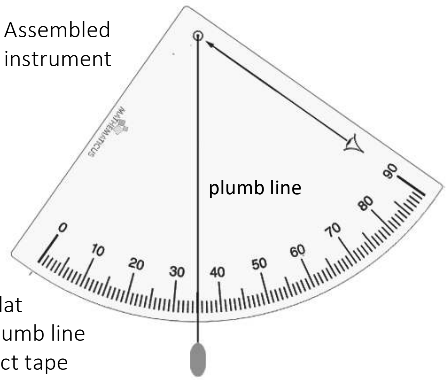
Fold upwards

Fold upwards



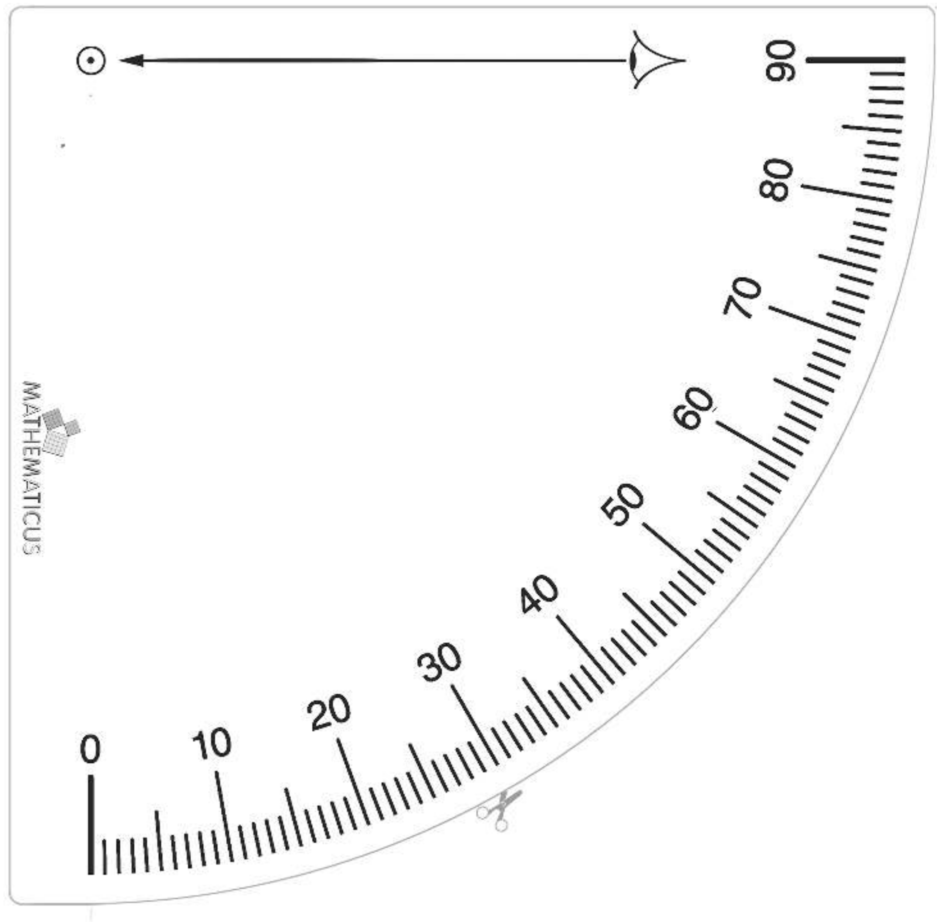
To get a printable high resolution electronic version of the instruments on this page and the Slide Rule used in the game, visit the DOWNLOAD section at **www.mathematicus.it**

Assembled instrument



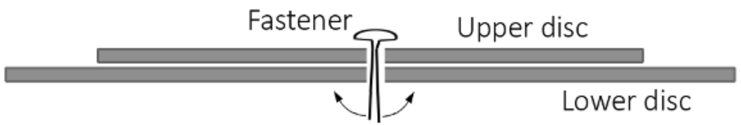
The Quadrant

Photocopy and paste the quadrant onto a sturdy flat sheet of cardboard and cut it out. Then attach a plumb line at the indicated point and secure a thread with duct tape on the reverse side. When pointed at a precise height, the height of the point on the horizon is indicated on the scale.



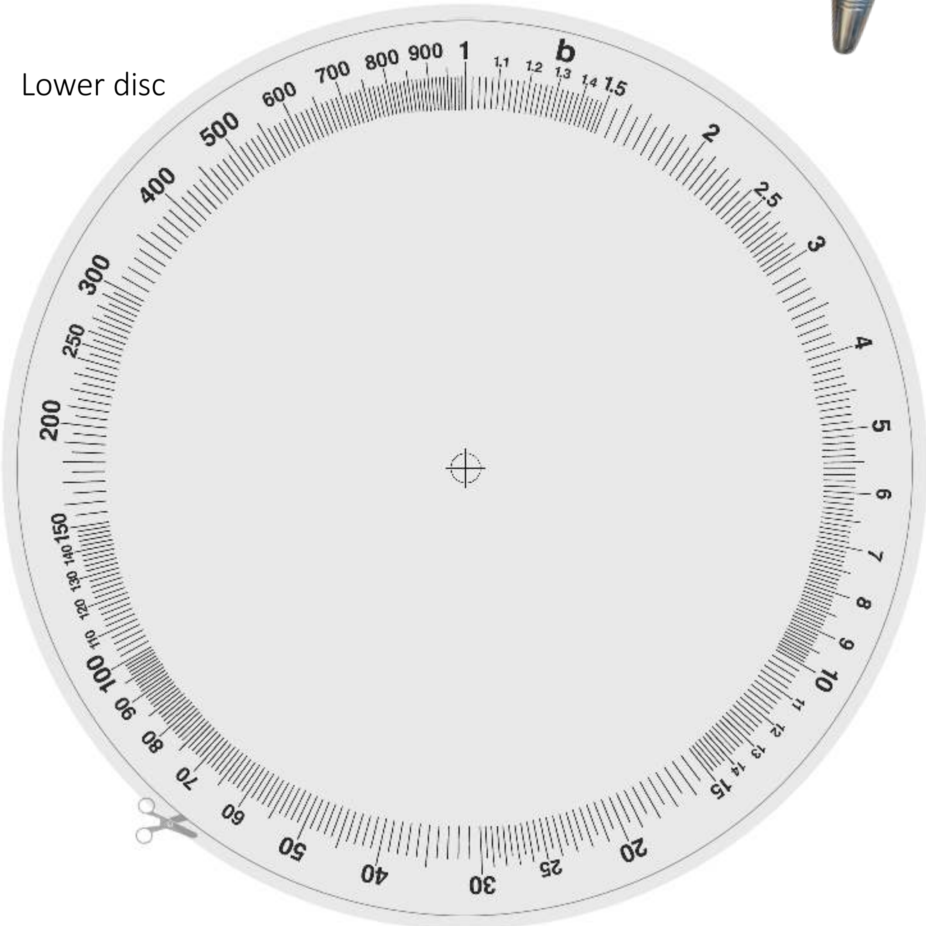
Right Angle Computer

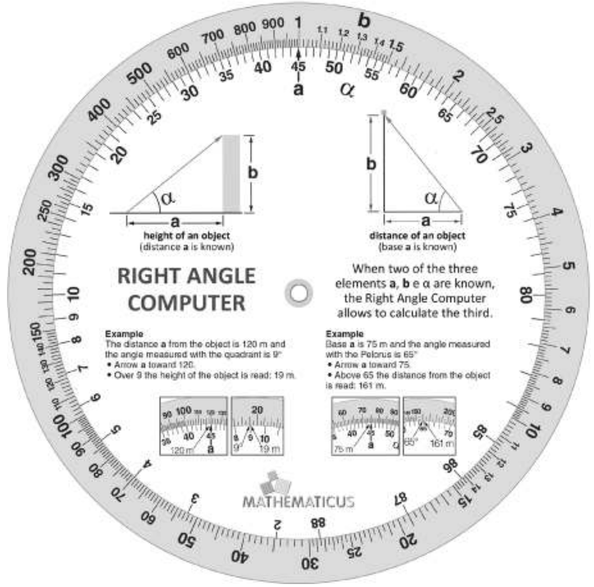
Photocopy and paste the two discs onto a sturdy flat sheet of cardboard, cut them out, pierce the central holes and put together with a rivet or paper fastener. Before making the hole, reinforce the area with one or two pieces of duct tape. A picture of the assembled instrument can be viewed on the next page.



A perforating tool should be used to precisely make the central hole. Place the perforator on the central point of the disk and, whilst holding it perpendicularly, gently tap the perforator with a hammer until the cardboard is pierced. A perforator used to make 5mm diameter holes is recommended with a rivet or fastener of the same diameter.

Lower disc





Assembled instrument

Upper disc